P425/1
PURE MATHEMATICS
Paper 1
3 hours

Uganda Advanced Certificate of Education

FACILITATION PAPER 2024

PURE MATHEMATICS

Paper 1

3 hours

Important Instructions:

Answer **all** the **eight** *questions in section* **A** *and any* **five** *questions from section* **B**.

Any additional question(s) answered will **not** *be marked.*

All *necessary working* ***must*** *be shown clearly.*

Begin each answer on a fresh sheet of paper.

Graph paper is provided.

Silent, non-programmable scientific calculators and mathematical tables with a list of formulae may be used.

 Turn Over

SECTION A: (40 MARKS)

1. Solve the equation $\sqrt{(2x+3)} - \sqrt{(x-2)} = 2$. *(05 marks)*

2. Evaluate $\int_0^{\pi/2} \sin^2\theta \cos^3\theta d\theta$. *(05 marks)*

3. A straight line AB of length 10 units is free to move with its ends on the axes. Find the locus of a point P on the line at a distance of 3 units from the end on the x-axis.

(05 marks)

4. Use Demoivre's theorem to find the square roots of $-2+2\sqrt{3}i$. *(05 marks)*

5. Solve the equation $\cos^2 2\theta + 5\sin^2 2\theta = 4$, for the range $0 \le \theta \le \frac{\pi}{2}$. *(05 marks)*

6. Use calculus of small changes to estimate $\log_2 7.95$. *(05 marks)*

7. Find the coordinates of the point where the line $\boldsymbol{r} = \begin{pmatrix} 5 \\ 0 \\ 5 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}$ meets the plane

$\boldsymbol{r} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = 11$. *(05 marks)*

8. The radius of a cylinder is increasing at the rate of 1 cm/min, while the height is increasing at the rate of 0.5 cm/min. Find the rate of change of the volume of the cylinder when the radius is 5 cm and the height is 10 cm. *(05 marks)*

SECTION B: (60 MARKS)

9. (a) Show that, if the equations $x^2+2x+a=0$ and $x^2+bx+3=0$ have a common root, then $(3-a)^2 = (6-ab)(b-2)$. *(05 marks)*

(b) The polynomial $p(x) = x^4 + Ax^3 + Bx^2 + Cx + 1$ is divisible by $(x+1)^2$ and leaves a remainder of 12 when divided by $x-1$. Find the values of A, B and C. *(07 marks)*

10. Given the curve $y = \dfrac{8}{x^2-4}$, determine the;

(a) coordinates of the turning point of the curve. *(03 marks)*

(b) equation of the asymptotes. Hence, sketch the curve. *(09 marks)*

11. (a) If x^3 and higher powers of x may be neglected, express $\left(1 + \frac{5x}{2} - \frac{3x^2}{2}\right)^8$ in the form $1 + ax + bx^2$. *(05 marks)*

(b) Find the first three terms in the expansion of $\frac{1}{\sqrt{4+x}}$ in ascending powers of x. Hence, obtain an approximation for $\frac{1}{\sqrt{4.16}}$. *(07 marks)*

12. (a) Given that $y = \sqrt{\left(\frac{\sin 3\theta}{1+\cos 3\theta}\right)}$, find $\frac{dy}{dx}$ in its simplest form. *(05 marks)*

(b) A curve is given parametrically by $x = 2t + 4t^3$, $y = t^2 + 3t^4$. Determine the coordinates of the points on the curve where $\frac{d^2y}{dx^2} = \frac{1}{14}$. *(07 marks)*

13. Consider the points $A(1, -1, 2)$ and $B(5, -1, -1)$.

(a) Find the equation of the line L through A and B. *(04 marks)*

(b) Find the equation of the plane perpendicular to L, and which passes through A. *(04 marks)*

(c) Find a point on L which is 20 units from A. *(04 marks)*

14. (a) Solve the equation $\sin 2x + \sin 3x + \sin 5x = 0$, for $0 < x < 360^0$. *(05 marks)*

(b) Show that $\frac{\cos(45^0+\theta)}{\cos(45^0-\theta)} = \frac{1-\tan\theta}{1-\tan\theta}$. Hence solve $4\cos(45^0+\theta) = \cos(45^0-\theta)$ for $0^0 < \theta < 90^0$. *(07 marks)*

15. Show that the line $y = mx+c$ touches the hyperbola $b^2x^2 - a^2y^2 = a^2b^2$ if $c^2 = a^2m^2 - b^2$. Hence find the equations of the tangents to the hyperbola $9x^2 - 25y^2 = 225$ which are parallel to the line $x - y = 0$. *(12 marks)*

16. Water is being heated in a kettle. At any time t seconds, the temperature of the water is $\theta^0 C$. The rate of increase of the temperature of water is at any time t is given by the differential equation

$$\frac{d\theta}{dt} = \lambda(120-\theta), \quad \theta \leq 100.$$

λ is a constant. When $\theta = 20^0 C$, $\frac{d\theta}{dt} = 1^0\mathbb{S}$ and $t = 0$. The kettle switches off when $\theta = 100^0 c$

(a) Solve the differential equation and hence show that

$$\theta = 120 - 100e^{-0.01t}.$$ *(07 marks)*

(b) Find the time to the nearest second when the kettle switches off. *(05 marks)*

END